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MODELLING THE EURO-NAIRA EXCHANGE RATE VOLATILITY IN THE POST-2023 ERA: A COMPARATIVE STUDY

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ABSTRACT

Exchange rate unification by the Central Bank of Nigeria in June 2023 introduced significant changes to Nigerian foreign-exchange markets, leading to volatility and uncertainty in exchange rates. This study seeks to find the appropriate innovation distribution that is best suited for modelling and forecasting EUR-NGN exchange rate volatility in Nigeria after unification using GARCH-type models. The data on the EUR/NGN exchange rate from July 3, 2023, to March 31, 2026, obtained from the Central Bank of Nigeria, and logged returns were used. The statistical tests results confirmed non-normality of the return data due to positive skewness, the presence of excess kurtosis, and significant ARCH effects; therefore, appropriate modelling was required to handle heteroskedasticity and the fat-tailed and asymmetric behaviour of the return data. Three alternative models GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) were estimated using 5 different innovation distributions, namely: the Normal, Student's t, Generalized Error Distribution (GED), as well as Skewed Student's t and Skewed GED. Information criterion measures and out-of-sample accuracy measures were used for model evaluation. The EGARCH-GED provided the best in-sample modelling, validating the existence of asymmetry and fat-tail characteristics in EUR-NGN returns, and the EGARCH-Skewed Student's t provided the better forecast. In conclusion, the Euro-Nigeria exchange rate returns after 2023 exchange rate unification exhibit persistence, asymmetry, and non-normal distribution characteristics; the EGARCH-Skewed Student's t model is best suited for exchange rate volatility forecast of the current exchange rate environment of Nigeria.

Keywords: Exchange Rate Volatility; EUR/NGN Exchange Rate; EGARCH; Innovation Distribution; Volatility Forecasting

1. Introduction

Nigeria's foreign exchange market has experienced a profound structural shift. In June 2023, a dual-regime market with large divergences between official and parallel rates was collapsing as the official rate struggled to keep pace with the parallel rate, approximately ₦462 per USD versus close to ₦750 in early 2023 (Investors King, 2023). Under the premise of eliminating arbitrageurs, the CBN on June 14th unified all existing exchange rate regimes, operating henceforth based on exchange rates set through a willing-

buyer/willing-seller mechanism. Instead of producing more exchange rate stability, this reform significantly exacerbated already-developing turbulence in Nigeria's currency market (Ayodeji, 2025). The question of whether, or how much, the turbulence intensified is not confined to Nigeria; its trading relations with the EU (over 20% of Nigeria's global trade; EU GSP Hub, 2026) mean its value against the euro is critical for investment decision, hedging activities, and monetary policy success. Beyond those directly involved in trading in or with Nigeria, its effects could impact foreign investment decisions in Nigeria, portfolio allocation choices of investors, and the overall financial stability of Africa's second-largest economy. The GARCH-class models are well-established for studying and modelling time-varying volatility in exchange rate data, and they have proven their success in stock, currency and commodity markets (Atoi, 2014; Goyal & Arora, 2010; Asemota & Ekejiuba, 2017). Success, however, depends on both the specification of variance equations and on the choices of innovation distribution for the error term. But which of these distributions work for the EUR/NGN volatility dynamics during Nigeria's post-2023 period remained questionable.

2. Literature Review

No study has systematically evaluated alternative innovation distributions within asymmetric GARCH models for the post-2023 EUR/NGN exchange rate, the point at which Nigeria's data-generating process structurally shifted. The available literature have examined portions of this problem separately but never together. The first paradigm focuses on a prespecified chosen distribution with no comparative approach to test its validity, like Abdullah *et al.* (2017) whose focus is entirely on the t-distribution. The second approach uses a comparative method to ascertain the performance of the chosen distribution; like Ampadu *et al.* (2024) compare five different distributions: Normal, Student's t, GED, Skewed Student's t, and SGED based on Monte Carlo simulation for best fit. None of these have addressed the EUR/NGN exchange rate after Nigeria unified the foreign market. However, the second question of whether post-2023 change has had impact on exchange rate volatility under asymmetric GARCH specification is addressed but with emphasis solely on its persistence; using EGARCH, Ayodeji (2026) investigates whether the 2023 unification led to structural changes on the behaviour of Naira-USD exchange rate volatility, suggesting volatility initially spiked and it then declined due to market adaptation hence EGARCH proved itself useful by capturing such asymmetrical responses toward volatilities, he used a single choice distribution to study volatility without testing it. Investigated the post-unification rate using GJR-GARCH models in Nigeria's multi exchange rate system, Oshota & Olanrele (2026) observed increased volatility persistence, supporting the claim for models capable of capturing the non-linear behaviour of volatility in emerging market Economies without considering distribution alternatives and Fernandes & Stumphius (2026) explored the overall transition post-unification and revealed that market-based approach affected transition mechanisms but did not deal with modelling volatility. It is by no means a coincidence that only the post-2023 period was studied here; including data before and after the exchange rate regime change will imply non-stationarity parameters and also bias the volatility estimates. The missing aspect thus is to systematically and comparatively identify which innovation distribution for models like GARCH captures the EUR/NGN exchange rates' volatility under Nigeria's current system of market pricing of dollars.

This study address this gap by comparatively testing several distributions namely Normal, Student's t and Generalized Error Distribution (GED) in the context of models: Symmetric GARCH and asymmetric GARCH. It does not assume that all distributions fits an exchange rate; it compares performance on data

based on statistics from models, checks validity using diagnostics statistics as well as from out-of-sample performance. Hence supported by established literature that shows that performance measured by accuracy, volatility measures as well as risk measures vary significantly depending on the choice of distributions (Wilhelmsson, 2006; Ampadu *et al*., 2024), this study identifies the single most reliable distribution.

3. Material and Methods

This section covers the details of the method and procedures followed to analyse EUR/NGN exchange rate volatility. The method covers: data collection; data transformation and conditioning; diagnostic testing; specifying, estimating, and selecting models. These procedures follow conventional practices within the financial time series framework.

3.1 Source of data

The secondary data utilized in this analysis are from the Central Bank of Nigeria. These daily EUR/NGN exchange rate data were extracted on every business day (Mondays through Fridays) and cover 3rd July, 2023 to 31st March 2026 (a sample size of 677 observations)

3.2 Data Transformation

Since exchange rate data tend to exhibit a number of undesirable characteristics, such as strong persistence and exhibiting a scale effect, a crucial step to use in modelling is the transformation of the data to returns. Transforming to returns helps in scale removal (which is important in modelling and predicting exchange rate values). More importantly, it helps in normalizing the variance of the series, making them more suitable to be the subject of statistical tests and models (Kumar,2013; Wong ,2017). Returns are given as

$$r_t = 100 \times \log\left(\frac{P_t}{P_{t-1}}\right) \quad (3.1)$$

where P_t = EUR/NGN rate at time t, P_{t-1} = EUR/NGN at time t-1. This returns generation method involved taking the natural logarithmic differences of observations multiplied by 100.

3.3 Stationarity Testing

The ADF test (Augmented Dickey-Fuller), as introduced by Dickey and Fuller (1979), is used to check the stationarity of EUR/NGN returns data. However, because of the concern for serial correlation present in exchange rates, the Phillips-Perron (PP) non-parametric test by Phillips & Perron (1988) is also employed. Test equation of ADF:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t \quad (3.2)$$

where y_t is the return series, Δ is the first difference operator, α a constant, βt a deterministic time trend (optional), and p is the number of lagged difference terms chosen to whiten the residuals ε_t.

The PP test is a non-parametric alternative to the ADF test. Instead of adding lagged difference terms to correct for serial correlation, it uses a kernel-based correction of the test statistic to account for heteroskedasticity and autocorrelation in the errors. Test regression (same as the original Dickey-Fuller):

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \varepsilon_t \quad (3.3)$$

The main difference between ADF and PP is in how autocorrelation and heteroskedasticity is accounted for; for instance, PP accounts for this by utilizing HAC corrections of the test statistic.

3.4 Preliminary Statistical Analysis

Compute descriptive statistics for the data to describe the basic properties. This includes Mean, Standard Deviation, Minimum, Maximum, Skewness, and Kurtosis.

The Jarque-Bera Test: Since the innovation distribution plays a very critical role in most of our models, it would be important at this preliminary stage to ensure that our returns data are normally distributed. That is, whether we need to use a t-distribution or not when constructing the parameters of the model. This can be done using the Jarque-Bera test statistic given by

$$JB = \frac{n}{6}\left[S^2 + \frac{(K-3)^2}{4}\right] \quad (3.4)$$

Where, n=sample size, S=sample skewness, and K=sample kurtosis

ARCH-LM Test Engle (1982): It is of critical importance that any time series used, especially the exchange rate time series, should be analyzed for the existence of ARCH components before any GARCH-family model is applied. This test is achieved using the Lagrange multiplier test, a commonly employed technique designed to test for ARCH disturbances, introduced by Engle(1982). In using the ARCH-LM test, there will be two stages. First, estimate the mean equation for a returns series (e.g. An ARIMA model), extract the residuals $(\hat{\varepsilon}_t)$. The squared residuals are then regressed on their own lagged values according to the auxiliary regression (Engle,1982):

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\varepsilon}_{t-i}^2 + u_t \quad (3.5)$$

where $\hat{\varepsilon}_t^2$ = squared residual at time t; α_0 = intercept term; α_i = coefficient associated with the i^{th} lag of the squared residuals; q = number of lagged squared residuals included in the test; u_t = random error term.

The null hypothesis of the ARCH-LM test is that there are no ARCH effects present in the residuals; that is, the variance is constant across time. This test can lead to either retaining or rejecting the null hypothesis. If the p-value associated with the test is less than 5% (the significance level), the null hypothesis can be rejected; this confirms the presence of ARCH components that can be modelled. However, if this test has a p-value higher than 5%, there is no evidence to reject the null hypothesis. Overall, the preliminary statistical analysis establishes whether the data satisfy the assumptions necessary for volatility modelling and determines the extent to which conventional single-regime models are likely to be adequate for describing the dynamics of the NGN/CNY exchange rate.

3.5 Conditional Mean and Conditional Variance

conditional mean is specified as

$$r_t = \mu_t + \varepsilon_t \quad (3.6)$$

where r_t= denotes the observed return, μ_t = is the conditional mean estimated using an ARIMA process, ε_t = is the innovation term. The innovation process is assumed to satisfy $\varepsilon_t = \sigma_t z_t$ where $z_t \sim IID(0,1)$ and (σ_t^2) represents the conditional variance, (z_t) denotes independently and identically distributed standardized innovations. The candidate model is compared via AIC, BIC, and residual diagnostics. The selected mean equation was used consistently across all subsequent variance specifications so that estimated volatility reflects genuine heteroskedasticity rather than residual serial dependence.

The conditional variance equation is expressed as

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (3.7)$$

GARCH(1,1) (Bollerslev, 1986)

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (3.8)$$

where: σ_t^2 is the conditional variance, ω= long-run variance component ,α= ARCH effect (shock impact) , β = GARCH effect (volatility persistence), $w > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ ensure positive variance $\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1$ ensures covariance stationarity. The ARCH effect calculates the impact of yesterday's shock, while the GARCH component obtains volatility persistence. Standard GARCH assumes that positive and negative shocks of equal absolute size increase σ_t^2 by exactly α.

GJR-GARCH(1,1) (Glosten *et al*., 1993)

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, I_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.9)$$

Where other variables are as previously defined, γ -asymmetry parameter measures leverage effects. A negative shock raises variance by $(\alpha+\gamma)$, a positive shock only by α. If $\gamma > 0$, bad news increases volatility more (leverage effect).

EGARCH(1,1) (Nelson, 1991)

$$\ln(\sigma_t^2) = \omega + \alpha\left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\ln(\sigma_{t-1}^2) \quad (3.10)$$

The term γz_{t-1} (with $z_{t-1} = \varepsilon_{t-1}/\sigma_{t-1}$) captures leverage. Typically, if $\gamma < 0$, a negative z_{t-1} adds a positive contribution ($\gamma \times$ negative $=$ positive), so volatility rises more after bad news.

The term $\alpha(|\, z_{t-1}\,| - \mathbb{E}\,|\, z_{t-1}\,|)$ gives a size effect symmetric in absolute value

3.6 Comparison Models

The purpose of comparison is to identify the best model; the comparison is made up of in-sample comparison and out-of-sample comparison approaches. The in-sample model comparison assesses how well the model fits the data that are used for estimation; it is carried out in order to identify the goodness of fit of the model. In this approach, all the observations are used to estimate the model and then assess with those observations. In this study, 677 observations, which are all the observations in the studies would be use to determine the in-sample comparison. In the in-sample comparison, the information criterion and log-likelihood are involved. The likelihood function measures how well a parameter set fits the data set estimated, higher log-likelihood is preferred. Information criteria are statistics designed to compare multiple models while considering both the goodness of fit and the degree of the model's complexity.

Akaike Information Criterion (AIC) :

$$AIC = -2LL + 2k \quad (3.11)$$

where: LL= log-likelihood , k= number of parameters

Bayesian Information Criterion (BIC):

$$BIC = -2LL + k\ln(n) \quad (3.12)$$

where: n= sample size. BIC penalizes model complexity more heavily than AIC. The smallest information criterion is preferred.

Out-of-sample forecasting of forecast accuracy using observations which were not included in the estimation set: One test set (80%) and one validation set (20%) based on observed data, and to forecast accuracy of the model, we use the validation set and we use test set to estimate a model. A model that forecasts better than one that just fits to past observations is a good thing.

Forecast Evaluation Measure

$$RMSE = \sqrt{\frac{1}{n}\sum(y_t - \hat{y}_t)^2} \quad (3.13)$$

where: y_t= realized volatility , $\hat{y}_t$= forecast volatility

Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum | y_t - \hat{y}_t | \quad (3.14)$$

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100}{n}\sum | \frac{y_t - \hat{y}_t}{y_t} | \quad (3.15)$$

Lower forecast evaluation measures shows good forecast

4. Results and Discussion

Results and discussion are presented in this part. Statistical calculations and analyses were all performed using statistical software R 4.5.2. The daily EUR/NGN exchange rate time series plot from July 2023 to March 2026 provides an initial preview of the dynamics underlying volatilities, which inspires this study.

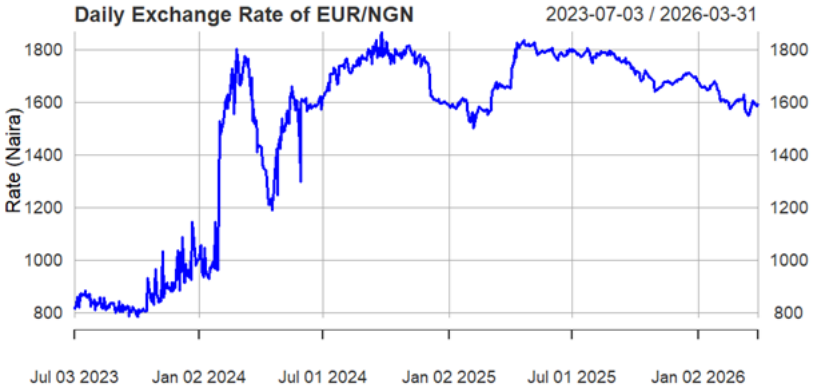


Figure 4.1: Daily Exchange Rate of EUR/NGN

By inspection of Figure 4.1, the phenomenon of volatility clustering - the "stylized fact" which GARCH family models attempt to measure- is evident. This confirms the justification for using the GARCH family instead of fixed variance based models. The crude prices series appears to be a representation of an auction that has abandoned any 'artificial anchors' and started to display the persistence of shocks and asymmetry typically found in emerging-market floating exchange rates.

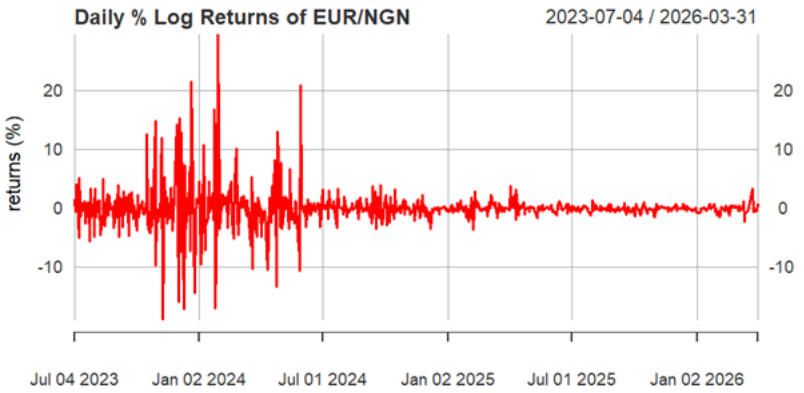


Figure 4.2: Daily % Log Returns of EUR/NGN

The daily log-return series for EUR/NGN offers a much sharper econometric picture than the price-level plot, and it directly visualizes several of the stylized facts that motivate a GARCH-family modelling approach

Descriptive statistics are given in Table 4.1

Table 4.1: Descriptive Statistics of Daily EUR/NGN

Statistic	EUR/NGN Series Returns	
Mean	1508.838	0.0998
Standard Deviation	342.3743	3.4278
Skewness	-1.1653	1.4444
Kurtosis	-0.2786	19.177
Minimum	783.3354	-18.8359
Maximum	1868.059	29.531
Q1 (25%)	1479.256	-0.6047
Median (50%)	1651.585	-0.0064
Q3 (75%)	1760.319	0.5628

From the descriptive statistics, the exchange rate series (EUR/NGN) was found to have a mean of 1508.84 and a relatively high standard deviation of 342.37 over the sample of March 2005-April 2006, which gives information on a good degree of variation in the sample. The level series was found to be negatively skewed (-1.165) and platykurtic (kurtosis was 0.279), and therefore departs significantly from normality. For the return series; the sample average of daily return was found to be 0.0998 and a standard deviation of 3.4278 which is evidence of high volatility of exchange rates, the return series was found to be heavily positively skewed (1.4444) and heavily leptokurtic (kurtosis was 19.177) again, it does not conform with normality and hence a strong case can be made for the use of GARCH family of models with assumption of heavy-tailed and skewed innovation distributions such as Student t distribution, GED distribution, skewed student t distribution and skewed GED distribution, and NOT normal distribution.

The results also imply that we may be required to employ asymmetric volatility models such as EGARCH and GJR-GARCH to account for possibly asymmetric effects of shocks to exchange rates. To verify

normality of the return series after concluding normality from descriptive statistics, a normality test was conducted. This is provided in Table 4.2.

Table 4.2 Test of normality

Test	Statistic	df/W	p-value	Interpretation
Jarque-Bera	10,594	2	< 2.2e-16	Strong rejection of normality

These strong Jarque-Bera normality tests provide enough evidence against a normal distribution of the EUR/NGN return series. They reject the null hypothesis that the return series are normally distributed since the Jarque-Bera statistic obtained are of 10,594 ($p < 0.001$). These results are therefore in line with those obtained from the descriptive statistics that show positive skewness of 1.4444 and an exceedingly high kurtosis (19.177), as can be observed in the tables.

In particular, it may be concluded from the test that the return distribution is skewed in one direction and heavy-tailed, with increased possibilities of extreme exchange rate movements compared to a normal distribution. The above pattern has also been empirically confirmed in the case of the financial return series and suggests the application of GARCH models of volatility with innovations that could follow a distribution more heavily tailed and skewed than Gaussian, such as Student's t distribution, GED, skewed Student's t, or skewed GED. As a result, biased volatility estimates and underestimation of market risk may result from assuming the return distribution to be normal.

Table 4.3 ARCH Test

ARCH LM-test; Null hypothesis: no ARCH effects
data: resid_Emean
Chi-squared = 171.43, df = 12, p-value < 2.2e-16

An ARCH-LM test was used to see whether conditional heteroskedasticity exists in the EUR/NGN return series. As we get a chi-square statistic of 171.43 with 12 degrees of freedom and a p-value less than 0.001, the null hypothesis of no ARCH effects should be rejected. Therefore, we conclude that the time-varying variance appears to exist for returns and is dependent on shocks that occurred in the previous period; volatility clustering is confirmed. Thus, the behaviour states that high conditional volatility days will be followed by another high conditional volatility day and low conditional volatility days will be followed by another low conditional volatility day; it is a known phenomenon on financial and exchange rate data. Standard classical models assuming constant variance will not be appropriate enough; the presence of the ARCH effect therefore suggests the possibility of modeling dynamic volatility with GARCH models. In conclusion, along with skewness, excess kurtosis and non-normality evidence strongly suggests estimating GARCH models using a heavy-tailed and skewed distribution for better forecasting of the volatility and risk management. After confirming the models as GARCH type model, question has risen is: which model fits the data well, and which model can forecast well.

Table 4.4: In-sample model comparison

Model	Distribution	LogLikelihood	AIC	BIC
eGARCH	Ged	-1096.506	4.083355	4.131039

Model	Distribution	LogLikelihood	AIC	BIC
eGARCH	Std	-1097.066	4.085430	4.133114
eGARCH	Sstd	-1096.770	4.088036	4.143668
gjrGARCH	Ged	-1099.412	4.094119	4.141804
sGARCH	Ged	-1101.206	4.097057	4.136794
gjrGARCH	Sged	-1099.385	4.097723	4.153354
sGARCH	Sged	-1101.195	4.100724	4.148408
gjrGARCH	Std	-1105.144	4.115348	4.163032
sGARCH	Std	-1106.930	4.118258	4.157995
gjrGARCH	Sstd	-1104.991	4.118486	4.174118
sGARCH	Sstd	-1106.929	4.121961	4.169645
eGARCH	Norm	-1165.662	4.335786	4.375523
gjrGARCH	Norm	-1173.827	4.366027	4.405764
eGARCH	Sged	-1172.375	4.368056	4.423687
sGARCH	Norm	-1177.941	4.377558	4.409347

To select the best specification, Table 4.4 compares three GARCH-family models such as GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) estimated using 5 different distributions of innovations (Normal, std t, ged, sstd and sged). Model adequacy was evaluated using Log-Likelihood (LL), AIC and BIC. Better model specification is reflected in the smallest values of AIC and BIC values. Among these 15 models, EGARCH(1,1) with GED distribution shows the best fit on the EUR/NGN exchange-rate return series and gives the lowest AIC (4.0834) and BIC (4.1310), closely followed by EGARCH(1,1) with Student's t distribution with AIC=4.0854 and BIC=4.1331. Therefore, it can be observed that returns for EUR/NGN exhibit not only asymmetry but also fat tails; indeed, incorporating leverage effects, the best fits were found when using non-Gaussian innovations. The success of EGARCH in fitting the data confirms previous findings showing that exchange-rate return are response asymmetrically to shocks. Indeed, Ayodeji (2025) found that the EGARCH models provided superiority compared to the GARCH model, as it accounts for the non-linear and asymmetrical volatilities of most exchange rate markets, especially emerging-market rates. Likewise, Adams & Uchema (2024) proved that EGARCH(1,1) was capable of capturing an asymmetrical behaviour of volatility for Nigeria exchange rate market and resulted in improved forecast ability more than other specified GARCH forms. These results also confirm Dinga *et al* (2023), which showed that EGARCH models capture persistence and asymmetry more effectively than any other standard GARCH models over all developed and developing currency markets. The high performance of GED suggests there's presence of extra kurtosis, i.e. Frequence extreme movements, in the EUR/NGN exchange rate return. Non-Gaussian distributions are usually perform better than the normal distribution due to their ability to handle large exchange rate changes (Dritsaki, 2019; Charfi & Mselmi, 2022). Since forecasting is a major goal for GARCH models to fit series, out-of-sample evaluation has been carried out as suggested by Hyndman & Athanasopoulos (2021) with an 80% training sample and a 20% test sample dataset. Out-of-sample forecasts of selected models fitted by 5 distributions were tested against different volatility measures, and these results are presented in Table 4.5:

Table 4.5: Out-of-sample Forecast Evaluation

Model	Distribution	RMSE	MAE	MAPE	QLIKE
eGARCH	sstd	0.6183762	0.4370302	19606.040	0.862105828
gjrGARCH	sstd	0.6183890	0.4366936	21600.257	0.083569705
eGARCH	norm	0.6184458	0.4365793	22116.063	-0.012363451
gjrGARCH	std	0.6188473	0.4370035	18947.665	0.083181914
eGARCH	std	0.6189642	0.4369647	15604.456	0.327472224
sGARCH	sstd	0.6190011	0.4363451	14752.689	0.095291419
sGARCH	std	0.6190447	0.4368269	16917.026	0.095252186
sGARCH	sged	0.6190574	0.4374931	11812.121	0.023743925
gjrGARCH	sged	0.6192916	0.4388011	12105.615	0.031020976
gjrGARCH	ged	0.6196650	0.4384291	7735.141	0.034986471
gjrGARCH	norm	0.6197135	0.4369898	18004.050	-0.022185824
sGARCH	ged	0.6198297	0.4385091	5771.920	0.024080431
sGARCH	norm	0.6204392	0.4365692	7515.594	-0.041091420
eGARCH	ged	0.6258612	0.4455985	2134.690	-0.002105105

Table 4.5 presents the out-of-sample forecasting performance of the estimated GARCH-family models under alternative innovation distributions. Forecast accuracy was assessed using Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Quasi-Likelihood Loss (QLIKE), where lower values indicate superior forecasting performance. The RMSE results show that EGARCH(1,1)-Skewed Student's t (sstd) achieved the lowest forecast error (RMSE = 0.618376), followed closely by GJR-GARCH(1,1)-sstd (RMSE = 0.618389) and EGARCH(1,1)-Normal (RMSE = 0.618446). This indicates that incorporating asymmetry, skewness, and heavy-tailed characteristics improves exchange-rate volatility forecasting. The finding supports Barguellil & Alnabulsi (2026), who reported that skewed heavy-tailed distributions often outperform symmetric distributions because financial returns commonly exhibit excess kurtosis and asymmetric behaviour.

Using MAE, the sGARCH(1,1)-sstd model produced the lowest value (0.436345), followed by EGARCH(1,1)-Normal and sGARCH(1,1)-Normal. The variation across forecasting measures confirms that no single model dominates all evaluation criteria, which is common in volatility modelling because RMSE places greater emphasis on large forecasting errors than MAE (Brooks, 2014). The MAPE results show that EGARCH(1,1)-GED recorded the lowest percentage error (2134.690), indicating that GED innovations provide improved proportional forecasting accuracy by capturing extreme movements and tail risk in EUR/NGN returns.

The QLIKE results show that EGARCH(1,1)-Normal achieved the lowest value (-0.0124), suggesting that normal-distribution models can still provide stable volatility forecasts despite weaker in-sample performance. This agrees with Daboh *et al* (2025), who noted that models with superior in-sample

fit do not always generate the best out-of-sample forecasts. The divergence between estimation and forecasting results further supports the view that historical fit and predictive accuracy should be evaluated separately (Brooks, 2019; Tsay, 2010).

Overall, EGARCH-GED provided the best in-sample fit, while EGARCH-sstd achieved the strongest forecasting performance. The result indicates that post-2023 EUR/NGN volatility is characterized by persistence, asymmetric shocks, skewness, and heavy tails. Therefore, EGARCH-sstd provides the most suitable framework for practical applications such as exchange-rate risk management, Value-at-Risk estimation, and volatility forecasting.

5. Conclusion

This study examined the optimal innovation distribution for modelling and forecasting post-2023 EUR/NGN exchange-rate volatility using GARCH(1,1), EGARCH(1,1), and GJR-GARCH(1,1) specifications under alternative distributional assumptions. Preliminary analyses revealed significant non-normality, excess kurtosis, positive skewness, and strong ARCH effects, confirming the suitability of GARCH-family models and heavy-tailed innovation distributions. The in-sample results showed that the EGARCH-GED specification achieved the highest log-likelihood and the lowest AIC and BIC values, indicating superior model fit and highlighting the importance of accounting for volatility asymmetry and fat-tailed return behavior. However, out-of-sample forecasting results revealed that the EGARCH-Skewed Student's t model produced the lowest forecast errors and the most accurate volatility forecasts. This divergence between in-sample and out-of-sample performance confirms that the model providing the best historical fit is not necessarily the model with the highest predictive accuracy. Overall, the findings suggest that EUR/NGN volatility under Nigeria's post-2023 exchange-rate regime is characterized by persistent, asymmetric, skewed, and heavy-tailed dynamics. Consequently, volatility forecasting, exchange-rate risk management, and Value-at-Risk estimation should rely on asymmetric GARCH models combined with flexible heavy-tailed distributions. The study therefore recommends the EGARCH-Skewed Student's t specification as the preferred forecasting model for post-unification EUR/NGN volatility and provides evidence that distributional choice remains a critical determinant of volatility modelling performance in emerging foreign-exchange markets.

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